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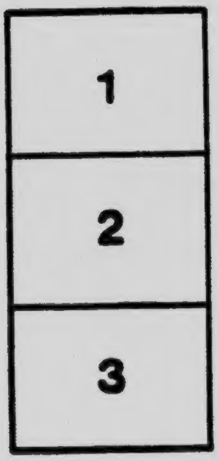

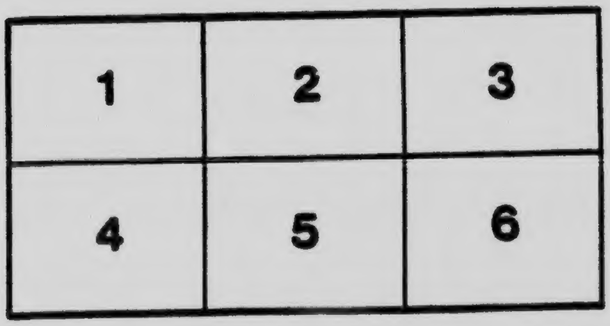

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THE ELEMENTS OF 93 LENOIS

BY

I. B. CANNON

THE ELEMENTS OF 93 LEONIS

By J. B. Cannon

THE star 93 Leonis ($\alpha = 11^h 43^m$; $\delta = +20^\circ 46'$) was announced to be a binary by Campbell and Wright in 1900,* from the measures of four plates taken by them in that year. It belongs to group *F*8 of Miss Cannon's classification. It was under observation here at three different periods in the years 1908, 1909, and 1910. During that time seventy-two plates were taken, the instruments used being the old and new single prism spectrographs.

The lines are not at all well defined and in many cases large differences result between the velocities given by the various lines and the mean of the plate. The lines appearing are chiefly due to iron, hydrogen, magnesium, titanium and carbon. Several lines of each element were measured but in the determination of the elements the titanium lines were discarded as so great differences between the velocities from its various lines existed that no dependence could be placed on them. A list follows giving the wave-lengths of the lines used and the element to which each is due :—

Wave-Length	Element	Wave-Length	Element
4861·527	*H*	4260·640	*C*
4549·766	*Fe*	4250·616	*Fe*
4481·400	*Mg*	4227·010	*Fe*
4415·301	*Fe*	4216·351	*Fe*
4404·927	*Fe*	4101·890	*H*
4352·006	*Mg, Cr*	4071·901	*Fe*
4340·634	*H*	4063·756	*Fe*
4325·939	*Fe*	4045·975	*Fe*
4271·760	*Fe*	3933·825	*Ca*

The period of oscillation was determined by the aid of the results obtained by Campbell and Wright from their measures of

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* *Astrophysical Journal*, Vol. XII., p. 255, 1900.

1900 and found to be 71·70 days. This was taken as being close enough, their observations being some forty periods away from the date of the first plate obtained here.

At the maximum of the curve the determination was somewhat unsatisfactory, the weather unfortunately being very bad at each return of the star to that part of its curve, so that only a few observations were obtained. Some time later a few plates may be taken to verify the results accepted.

The seventy-two observations were grouped into the following normal places :—

	Julian Date	Phase	Velocity	Residual O-C	Weight
1	2,418,317·48	37·28	− 20·2	+ 0·12	3
2	515·92	43·02	− 20·9	+ 1·25	2
3	497·50	48·93	− 16·1	+ 3·18	1·5
4	338·54	52·02	− 17·3	− 1·55	2
5	672·90	57·74	− 10·0	− 4·12	2
6	649·39	60·11	+ 1·5	+ 2·26	2
7	480·61	64·81	+ 10·1	+ 0·23	2·5
8	456·85	69·10	+ 22·9	+ 4·46	1
9	543·96	4·35	+ 22·0	− 3·13	2
10	550·70	9·61	+ 22·6	0·01	2
11	804·94	13·88	+ 19·1	+ 2·36	2
12	267·14	18·58	+ 8·0	+ 0·45	2
13	626·12	23·27	− 3·0	− 2·05	1
14	554·60	27·22	− 7·1	+ 0·04	2
15	601·07	30·09	− [illegible]	− 2·55	2

Dr. King's graphic method was then used in determining the preliminary elements. These were :—

$$\gamma = -0{\cdot}578 \text{ km.}$$
$$K = 24 \text{ km.}$$
$$e = {\cdot}1$$
$$\omega = 330^\circ$$
$$T = 2{,}418{,}028{\cdot}85$$
$$P = 71{\cdot}7 \text{ days}$$
$$\text{and } \Sigma pvv = 147{\cdot}1$$

It was then decided to apply the method of Least Squares to reduce the value Σpvv if possible. Observation equations were formed by means of the formula of Lehmann-Filhés.*

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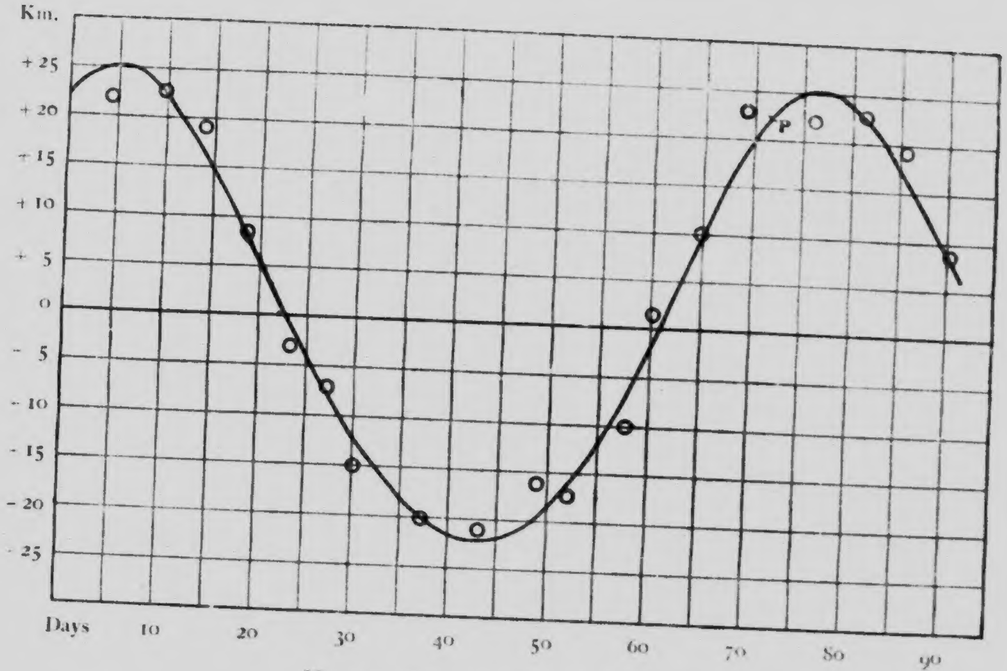
* *A. N.*, No 3242.



VELOCITY CURVE OF 93 LEONIS

From the observation equations there result the following normal equations :—

$$\begin{aligned}
29{\cdot}000x - 1{\cdot}182y - 2{\cdot}673z + {\cdot}426u - {\cdot}436v - 12{\cdot}375 &= 0\\
12{\cdot}975y - 2{\cdot}954z - {\cdot}632u + {\cdot}715v + 3{\cdot}205 &= 0\\
14{\cdot}556z - 2{\cdot}571u + 2{\cdot}620v + 5{\cdot}272 &= 0\\
15{\cdot}578u - 15{\cdot}517v + 4{\cdot}554 &= 0\\
15{\cdot}601v - 1{\cdot}601 &= 0
\end{aligned}$$

the solution of which gives

$$\begin{aligned}
x &= +{\cdot}3791\\
y &= -{\cdot}3354\\
z &= -{\cdot}4327\\
u &= +{\cdot}4741\\
v &= +{\cdot}8652.
\end{aligned}$$

Hence the corrections—

$$\begin{aligned}
\delta\gamma &= +{\cdot}379 \text{ km.}\\
\delta K &= -{\cdot}335 \text{ km.}\\
&\ -{\cdot}018\\
\delta\omega &= +1^{\circ}{\cdot}146\\
\delta T &= +{\cdot}405.
\end{aligned}$$

These results give satisfactory differences between the residuals from the observation equations and the computed residuals, the highest difference being ·06. Σvv was reduced to 137·4.

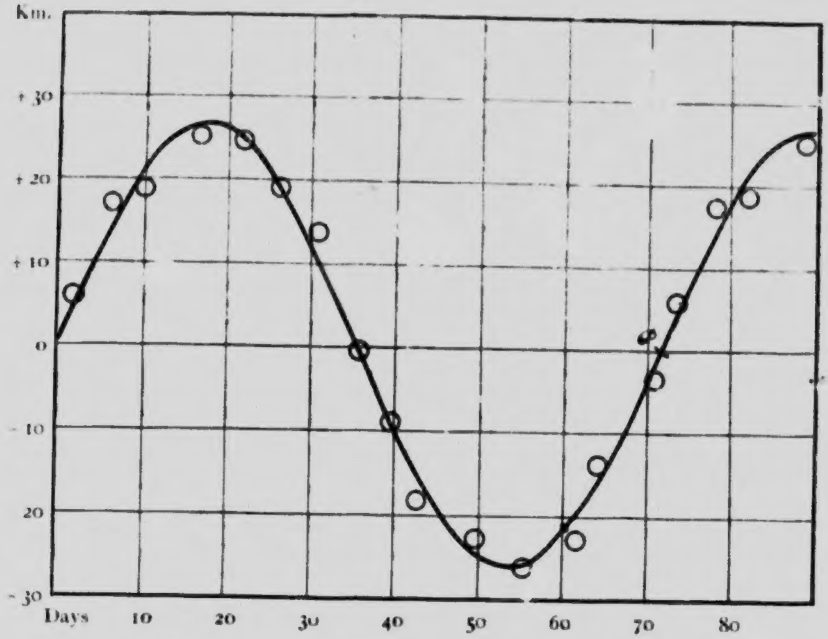
The probable error [illegible] a normal place was found to be ± 2·5 and that of an average p[illegible] — found by scaling residuals from the curve — to be ± 3·4.

Probable errors of the e[illegible] were also found and are given below, together with the fi[illegible] [illegible]ed.—

$$\begin{aligned}
\gamma &= - \text{[illegible] km.}\\
K &= 23{\cdot}6\text{[illegible] km.}\\
e &= {\cdot}082 \quad \text{[illegible]}\\
\omega &= 331^{\circ}\ \text{[illegible]}\\
T &= 2{,}4\text{[illegible]}\ \text{J. D.}\ \text{[illegible]}\\
P &= 71\ \text{[illegible] days}\\
a \sin i &= 23{,}2\text{[illegible]},000 \text{ km.}
\end{aligned}$$

VELOCITY CURVE OF 93 LEONIS

Thinking the probable error of a plate rather high for this type of star, it was decided to try measuring the plates with the spectro-comparator. All the plates, with the exception of ten which were too faint for this method, were remeasured. The standard plates used were numbers 3172 and 3755, the former a sky plate taken with the new single-prism spectrograph, and the latter a sun plate taken with the old.



Velocity Curve of 93 Leonis. — Comparator Measurements

The measures were grouped into fifteen normal places as before, each plate being placed in the same normal place as in the former work. These places are given below with the mean Julian day, the phase from final T, the mean velocity, weight, and residual from final curve.

NORMAL PLACES

	Julian Day	Phase	Velocity	Weight	Residuals O-C
1	2,418,317·48	49·83	− 22·7	5	+ 2·09
2	631·43	55·47	− 26·2	2	− 0·11
3	497·[illegible]	61·48	− 22·9	3	− 2·38
4	285·08	64·31	− 14·0	3	+ 1·75
5	672·90	70·97	− 3·5	4	2·32
6	649·52	1·76	+ 5·8	3	+ 1·11
7	480·61	6·32	+ 17·2	5	+ 2·57
8	456·85	10·22	+ 18·5	2	2·82
9	543·96	16·96	+ 25·0	3	− 1·66
10	558·42	21·89	+ 24·7	3	− 0·15
11	804·94	26·43	+ 18·9	3	− 0·30
12	190·95	39·85	+ 13·7	3	+ 2·77
13	590·48	36·03	− 0·4	2	+ 0·22
14	562·36	39·55	− 8·8	3	0·28
15	652·17	42·60	− 18·4	3	3·65

These normal places were plotted and the following preliminary elements were obtained graphically :—

$$\gamma = 0$$
$$K = 26 \text{ km.}$$
$$e = 0$$
$$P = 71{\cdot}70 \text{ days.}$$

The value of Σpvv was computed and found to be 207. One least squares solution was applied to the elements. Values of T and ω were assumed to be —

$$T = 2{,}418{,}088{\cdot}405 \text{ J. D.}$$
$$\text{and } \omega = 270^\circ$$

for the purposes of the solution. This value of T was taken as fixed, for with e zero and ω 270° it would be impossible to obtain corrections to e, ω and T as two of the equations would be identical.

Observation equations were formed and normal equations found from them as follows :—

$$\begin{array}{rrrrr}
47x & + \cdot183\,y & + 1{\cdot}657\,z & + 2{\cdot}498\,u & - 9{\cdot}340 = 0 \\
 & 21{\cdot}018\,y & - \cdot967\,z & + \cdot800\,u & - 11{\cdot}476 = 0 \\
 & & 23{\cdot}699\,z & + \cdot887\,u & - 5{\cdot}108 = 0 \\
 & & & 25{\cdot}985\,u & - 10{\cdot}570 = 0
\end{array}$$

The solution of these equations gave the following corrections to the elements :—

$$\begin{aligned} \delta\gamma &= + \cdot 17 \text{ km.} \\ \delta K &= + \cdot 54 \text{ ''} \\ \delta e &= + \cdot 008 \\ \delta\omega &= + 0^\circ \cdot 81. \end{aligned}$$

Hence the corrected values of the elements :—

$$\begin{aligned} \gamma &= + \cdot 17 \text{ km.} \\ K &= 26 \cdot 54 \text{ ''} \\ e &= \cdot 008 \\ \omega &= 270^\circ \cdot 81 \\ T &= 2{,}418{,}088 \cdot 405 \text{ J. D.} \\ P &= 71 \cdot 70 \text{ days.} \\ a \sin i &= 26{,}170{,}000 \text{ km.} \end{aligned}$$

These elements gave a new value of Σpvv of 196. This is a very small reduction — only about 5 per cent. — but the excellent agreement between the residuals — computed and observation equation — showed that further application of least squares would be useless.

The probable error of a normal place and of an average plate were computed and found to be ± 2·85 and ± 3·4 respectively. There were three plates, 1498, 1599 and 2134 which for some unknown reason gave abnormally large residuals, being − 14·0, + 11·6, and + 13·0 respectively. If these be omitted the probable error of an average plate becomes ± 2·87.

The probable error of the elements were also computed and are attached to the final values in the table below. The table gives the elements obtained in the two ways of measuring, together with their probable errors.

Element	Micrometer		Comparator	
γ	− ·20 km.	± ·31	+ ·17 km.	± ·42
K	23·665 km.	± ·48	20·54 km.	± ·62
e	·082	± ·02	·008	± ·02
ω	331°·15	± 10°·72	270·81	± 1°·26
T	2,418,029·255 J.D.	± 2d·1	2,418,088·405 J.D.	
P	71·70 days		71·70 days	
$a \sin i$	23,250,000 km.		26,170,000 km.	
Probable error of average plate		± 3·5		± 3·4 ± 2·0*
Probable error of average normal places		± 1·5		± 1·4

* Omitting plates, 1498, 1599, 2134.

Judging from the probable errors of an average plate, there is little difference between the two methods of measurement. There is something to be said in favor of each. Plates under exposed *may* be measured by the micrometer, which would not permit of measurement by the comparator. On the other hand it is impossible to obtain good agreement in the velocities obtained from the various lines in a spectrum like that of this star, and on such a spectrum, perhaps, the best work can be done with the comparator, which enables the measurer to strike a mean all along the plate.

As regards the elements obtained, *K* is the only one which shows a change worthy of note. The eccentricity is very small in both cases and the differences in ω need not be remarked upon.

In conclusion I wish to acknowledge my indebtedness to the Director for his interest and help in the work.

DOMINION OBSERVATORY,
OTTAWA, CANADA,
November 1910.